

Element! $\leftarrow$ Mix them up $\leftarrow$

1. Number Properties

Divisibility rules

Divisible by 2: Last digit is even (e.g. 124 $\rightarrow$ ends in 4)

Divisible by 3: Sum of digits is divisible by 3 (e.g. 123 $\rightarrow 1+2+3 = 6$)

Divisible by 4: Last two digits form a number divisible by 4 (e.g. 316 $\rightarrow 16 \div 4 = 4$)

Divisible by 5: Last digit is 0 or 5 (e.g. 85, 90)

Divisible by 6: Number is divisible by both 2 and 3 (e.g. 132 is even and $1+3+2 = 6$)

Divisible by 9: Sum of digits is divisible by 9 (e.g. 729 $\rightarrow 7+2+9 = 18$)

Divisibility and Neighboring Numbers

If x is divisible by a number $d > 1$, then $x + 1$ and $x - 1$ are not divisible by d .

Divisibility in Consecutive Integers

In n consecutive integers, at least one of them is divisible by n .

Factorial

$5! = 5*4*3*2*1$ or $1*2*3*4*5$

$5!$ Divisible by 3?

$5! - 1$ is even or odd? $\leftarrow$

even $\leftarrow$ divisible by 2 $\leftarrow$ can pull 2 out

$K = 5m \leftarrow 3K = 3*5m$

Parity rules

Even + Even = Even

Odd + Odd = Even

Even + Odd = Odd

Even $\times$ Anything = Even

Odd $\times$ Odd = Odd

Positive, negative, and zero rules

Positive $\times$ Positive = Positive

Positive $\times$ Negative = Negative

Negative $\times$ Negative = Positive

Any number $\times 0 = 0$

Properties of 0 and 1

Additive Identity (0):

Any number $+ 0 =$ That number

e.g. $5 + 0 = 5$

Negative of a Number:

$a + (-a) = 0$

e.g. $7 + (-7) = 0$

Multiplying by 0:

$a \times 0 = 0$

$0 \times a = 0$

e.g. $12 \times 0 = 0$

0 Divided by a Nonzero Number:

$0 \div a$ (where $a \neq 0$) = 0

e.g. $0 \div 5 = 0$

Division by 0 is Undefined:

$a \div 0$ is **undefined**

Zero Product Property:

If $a \times b = 0$, then $a = 0$ or $b = 0$ (or both)

e.g. If $xy = 0 \rightarrow$ either $x = 0$ or $y = 0$

Integer rules

Integer $+$ Integer = Integer

Integer $-$ Integer = Integer

Integer $\times$ Integer = Integer

Division of integers may not be an integer

Consecutive number rules

Consecutive integers differ by 1

Sum of n consecutive integers starting from k :

$$\text{Sum} = n/2 \times [2k + (n - 1)]$$

Sum of first n positive integers (1, 2, ..., n):

$$\text{Sum} = n(n + 1) / 2$$

Arithmetic sequence rules

An arithmetic sequence has a constant difference d between terms:

$$a_n = a_1 + (n - 1)d$$

Sum of first n terms:

$$S_n = n/2 \times [2a_1 + (n - 1)d]$$

or equivalently:

$$S_n = n/2 \times (a_1 + a_n)$$

Prime numbers

Only divisible by 1 and itself

2 is the only even prime

Greatest common divisor (GCD)

Largest number dividing two or more numbers

Least common multiple (LCM)

Smallest number that both or all numbers divide into

Remainder

If $y=qx+r$, then r is the remainder when y is divided by x

Linking remainder with decimals and mixed numbers

When dividing a by b :

- Quotient q is the whole number part
- Remainder r as fractional part r/x
- Division result as mixed number:

$$q\frac{r}{x}$$

- Or decimal by dividing remainder by divisor:
 $y \div x = q + r/x$ (decimal form)

Absolute value

Distance from zero on the number line

$$|x|=x \text{ if } x \geq 0$$

$$|x|=-x \text{ if } x < 0$$

Additional properties useful for GMAT and GRE

Zero is neither positive nor negative

Sum of two positive numbers = Positive

Sum of two negative numbers = Negative

Sum of positive and negative depends on which absolute value is greater

Multiplying or dividing both sides of an inequality by a negative reverses the inequality sign

Consecutive integers differ by 1

Absolute value inequalities:

$$\text{If } |x| < a, \text{ then } -a < x < a$$

$$\text{If } |x| > a, \text{ then } x < -a \text{ or } x > a$$

2. Fractions, Decimals, Percents

Fractions

- A fraction represents part of a whole: numerator $\div$ denominator
- Proper fraction: numerator $<$ denominator (less than 1)
- Improper fraction: numerator $\geq$ denominator (equal or greater than 1)
- Mixed number: whole number plus proper fraction (e.g., $2 \frac{1}{3}$)

Operations with fractions

- Addition/Subtraction: convert to common denominator, then add/subtract numerators
- Multiplication: multiply numerators $\times$ numerators, denominators $\times$ denominators
- Division: multiply by reciprocal of divisor
Example: $(a/b) \div (c/d) = (a/b) \times (d/c) = (a \times d) / (b \times c)$

Simplifying fractions

Divide numerator and denominator by their greatest common factor (GCF)

Decimals

- Decimal numbers represent fractions with denominators as powers of 10
- Convert fraction to decimal by division: numerator $\div$ denominator
- Terminating decimals: end after finite digits (e.g., 0.75)
- Repeating decimals: repeat a pattern indefinitely (e.g., 0.333...)

Link with Percents

- Percent means “per hundred”
- To convert percent to decimal, divide by 100 (e.g., $75\% = 0.75$)
- To convert decimal to percent, multiply by 100 (e.g., $0.25 = 25\%$)
- To convert fraction to percent, convert fraction to decimal first then multiply by 100

Percent increase/decrease

Percent change = $(\text{New value} - \text{Original value}) \div \text{Original value} \times 100\%$

- Increase if new > original
- Decrease if new < original

Common percent phrases

- “x% of y” means $(x \div 100) \times y$
- “What percent of y is x?” means $(x \div y) \times 100\%$

Method 1: Within-Ratio Comparison

(Internal Ratio Form)

- **Format:** $A : B = 8 : 5 \rightarrow A/B = 8/5$

Method 2: Across-Ratio Comparison

(Proportional Matching)

- **Format:** $A : B = 8 : 5 \rightarrow A/8 = B/5$

Percents Topic

Percent means “per hundred” and is a way to express ratios out of 100. For example, 25% means 25 out of 100 or $25/100$.

To convert:

- **Percent to decimal:** divide by 100
Example: $75\% = 75 \div 100 = 0.75$
- **Decimal to percent:** multiply by 100
Example: $0.25 = 0.25 \times 100 = 25\%$
- **Fraction to percent:** convert fraction to decimal first, then multiply by 100
Example: $3/4 = 0.75 = 75\%$

Percent increase

Value increase

Formula:

Percent increase = $[(\text{New value} - \text{Original value}) \div \text{Original value}] \times 100\%$

Example: If price increases from \$80 to \$100, percent increase = $[(100 - 80) \div 80] \times 100\% = 25\%$

Percent decrease

Value decrease

Formula:

Percent decrease = $[(\text{Original value} - \text{New value}) \div \text{Original value}] \times 100\%$

Example: If price decreases from \$80 to \$60, percent decrease = $[(80 - 60) \div 80] \times 100\% = 25\%$

Percent less than 1%

$1\% = 0.01$

Example: $0.5\% = 0.005$ in decimal form

Percent more than 100%

New value = Original value $\times$ (Percent $\div$ 100)

Example: 150% means 1.5 times the original value (50% increase)

Example 2: Original is 40 and increased by 150%, new value = $40 \times 1.5 = 60$